Double-layer shocks in a magnetized quantum plasma

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The formation of small but finite amplitude electrostatic shocks in the propagation of quantum ion-acoustic waves (QIAWs) obliquely to an external magnetic field is reported in a quantum electron-positron-ion (e-p-i) plasma. Such shocks are seen to have double-layer (DL) structures composed of the compressive and accompanying rarefactive slow-wave fronts. Existence of such DL shocks depends critically on the quantum coupling parameter H associated with the Bohm potential and the positron to electron density ratio δ. The profiles may, however, steepen initially and reach a steady state with a number of solitary waves in front of the shocks. Such novel DL shocks could be a good candidate for particle acceleration in intense laser-solid density plasma interaction experiments as well as in compact astrophysical objects, e.g., magnetized white dwarfs.

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Recent studies [1–4] have shown that the formation of stationary current free double layers (DLs) may be possible in dense plasma environments where the forces namely, (i) quantum statistical pressures for electrons and positrons, (ii) electron and positron tunneling associated with the Bohm potential play important roles in the propagation of quantum ion-acoustic waves (QIAWs). Inclusion of these quantum forces along with electron and positron angular momentum spin allows also the existence of very high-frequency dispersive electrostatic and electromagnetic waves (e.g., in the hard X-ray and γ-ray regimes) with extremely short wavelengths (see for recent review in quantum plasmas, Ref. [5]). However, there was no indication or explanation of the forward propagating (upstream) DLs that may exist, especially in a strongly magnetized quantum plasma, as a form of compressive and accompanying rarefactive slow-wave fronts. Formation of these upstream DLs is one of the most striking features of the magnetohydrodynamic (MHD) shocks, and has been observed by Tajima et al [6], possibly for the first time, in a classical MHD flow. Moreover, there is also a wide-spread interest in investigating DLs as a possible acceleration mechanism in various space and astrophysical plasma environments (see, e.g., [7–11]). In a thought-provoking series of discussions, Alfvén considered DLs to be a central paradigm in plasma astrophysics [12].

Our current knowledge of DLs in plasma physics, however, is insufficient for us to judge with much confidence and belief what roles DLs may, indeed, play especially in astrophysical environments. Thanks for the evidence of particle acceleration in a magnetized white dwarf which has been reviewed by Jager in the past [13]. It has been reported there that if the current density is large enough in a tenuous magnetosphere, DLs can be formed leading to a large electric field, and hence monoenergetic electrons and ions with higher energies. Recently, a new study (see, e.g., Ref. [14]) demonstrates that a white dwarf star may pulse like a pulsar, e.g., AE Aquarii can emit pulses of high energy X-rays as it rotates on its own axis. It has also been predicted that since pulsars are known to be sources of cosmic rays, white dwarfs should be quiet but numerous particle accelerators [15], contributing many of the low-energy cosmic rays in our galaxy. One may thus be interested to know the physical mechanisms of such particle accelerations in these astrophysical compact objects. In this regard it may be noted that ever since the discovery of cosmic rays, the problem of understanding their origin and acceleration has not yet been fully understood. In this context, a number of models and several processes have been proposed in the early days for their origin and the acceleration mechanism (see for some discussions, e.g., Ref. [16]). Furthermore, in addition to the shock-wave acceleration mechanism proposed early by Colgate and white [17], Goldreich et al [18] and Gunn et al [19] had also shown that pulsars can accelerate particle to very high energies. Recently, the expected abundance of cosmic-ray electrons and positrons from pulsars and magnetars have been studied [20].

A basic prerequisite for particle acceleration in pulsars is the presence of magnetospeheric plasma. On the basis of which Goldreich and Julien [18] had shown that the large electric field due to a nonzero $E.B$ at the neutron star surface, can overcome the gravitational potential and ensure a minimum plasma density. They also showed that the plasma flowing along the co-rotating magnetic field lines and escaping out at the light cylinder (defined by the distance from the center of the neutron star) will experience a potential drop and hence be accelerated. Furthermore, since an oblique magnetic rotator (in which the magnetic and rotational axes do not coincide) emits electromagnetic (EM) waves of the same frequency as that of rotation, particles can be accelerated at or beyond the light cylinder to high energies extremely efficiently by

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the low-frequency EM waves emitted by a pulsar. This was first proposed by Gunn et al [19].

Note, however, that in addition to degenerate electrons, there would also exist degenerate positrons, e.g., in magnetars and in the next generation of intense laser-solid density plasma interaction experiments [5] for which DLs could well be responsible for ion acceleration therein.

Motivated by these facts, we report in this work, the existence of forward propagating slow-wave shocks in a magnetized quantum plasma, whose constituents are electrons, positrons and positive ions (hereafter referred as e-p-i). Because of the sufficient lifetime of the positrons compared to the ion time scale, plasma can become an admixture of electrons, positrons and ions. Such e-p-i plasmas are believed to exist, e.g., in the magnetosphere of pulsars, in the active galactic nuclei, in the regions of the accretion disks surrounding the central black holes [18, 21–23], Van Allen radiation belts, near the polar cap of fast rotating neutron stars [24], supernova remnants [25], in intense laser fields [26], in compact astrophysical objects (e.g., giant planetary interiors, white dwarfs, neutron stars/ magnetars) [27], in tokamaks [28] as well as in the early universe [29]. Note that the process of electron-positron (e-p) pair creation and annihilation may occur in relativistic plasmas at high temperatures, when the temperature of the plasma exceeds the rest mass of electrons. However, for the propagation of ion-acoustic waves in e-p-i plasmas, the e-p pair annihilation can be neglected in the sense that the electron-positron lifetime is much larger than the characteristic time scale for collective oscillations (see for detail derivation and discussion, e.g., Ref. [30]).

In what follows, we will consider the quantum force associated with the Bohm potential to provide higher order dispersion along with the charge separation effect as well as the magnetic-field-induced dispersion anisotropy. The ions are assumed to be cold and the motion is considered on the ion-acoustic time scale. Because of their light masses, electrons and positrons will be highly magnetized compared to the ions (ion Larmor radius is much larger than that of electrons or positrons), and will move almost parallel to the external magnetic field, so that electrons and positrons may be described by the quantum modified Boltzmann-like distributions. We will derive a modified Korteweg-de Vries (MKdV) equation with a quadratic as well as cubic nonlinearity that describes the dynamics of quantum ion-acoustic (QIA) DLs, and investigate some interesting properties of such DLs for large times. We observe a novel DL structure to the upstream shocks, which is composed of a compressive slow-wave shock and a rarefactive slow-wave front.

Under the above assumptions, the motions of ions, electrons and positrons in the propagation of QIAWs obliquely to the external magnetic field $B = B_0\hat{z}$ can be described by the following equations [31].

$$\partial_t n_i + \nabla.(n_i \mathbf{v}) = 0, \tag{1}$$

$$(\partial_t + \mathbf{v}.\nabla)\,\mathbf{v} = -\nabla\phi + \omega_c \mathbf{v}\times\hat{z}, \tag{2}$$

$$n_e^{2/3} = 1 + 2\phi + \frac{H^2}{\sqrt{n_e}}\nabla^2\sqrt{n_e}, \tag{3}$$

$$n_p^{2/3} = 1 - 2\delta^{-2/3}\phi + \frac{H^2}{\sqrt{n_p}}\nabla^2\sqrt{n_p}, \tag{4}$$

$$\nabla^2\phi = n_e - \delta n_p - (1-\delta)\, n_i, \tag{5}$$

where n_α is the number density of $\alpha-$species particle normalized by the unperturbed value $n_{\alpha 0}$, $v \equiv (v_x, v_y, v_z)$ is the ion fluid velocity normalized by the ion-acoustic speed $c_s = \sqrt{k_B T_{Fe}/m_i}$ with k_B denoting the Boltzmann constant, T_{Fe} the electron Fermi temperature and m_i the ion mass. Also, ϕ is the electrostatic potential normalized by $k_B T_{Fe}/e$, $\omega_c = eB_0/m_i\omega_{pi}$ is the ion-cyclotron frequency normalized by the ion plasma frequency, $\omega_{pi} = \sqrt{n_{i0}e^2/\epsilon_0 m_i}$, $\delta = n_{p0}/n_{e0}$ is the positron to electron density ratio, $H = \hbar\omega_{pe}/k_B T_{Fe}$ is the quantum parameter denoting the ratio of the electron plasmon energy density to the Fermi thermal energy. The space and time variables are normalized by the Fermi Debye length $\lambda_F = c_s/\omega_{pi}$ and the ion plasma period ω_{pi}^{-1} respectively. In Eqs. (3), (4) we have used the following Fermi-Dirac pressure law due to electron and positron degeneracy [32].

$$P_\alpha = \frac{1}{5}\frac{m_\alpha V_{F\alpha}^2}{n_{\alpha 0}^{2/3}} n_\alpha^{5/3}, \tag{6}$$

where $V_{F\alpha} = \sqrt{k_B T_{F\alpha}/m_\alpha}$ is the Fermi thermal speed. In Eqs. (3), (4), the terms proportional to H associated with the Bohm potential, account for typical quantum effects such as tunneling. In a broad sense, we refer to these particularities arising from the wave-like nature of the charge carriers as 'quantum diffraction effects'.

In order to describe now the dynamics of propagating DLs, we employ a reductive perturbation technique in which the independent variables are stretched as $\xi = \epsilon(l_x x + l_y y + l_z z - Mt)$, $\tau = \epsilon^3 t$, where ϵ is a small parameter representing the strength of the wave amplitude, M is the phase speed normalized by c_s and l_x, l_y, l_z are the direction cosines of the wave vector along the axes such that $l_x^2 + l_y^2 + l_z^2 = 1$. The dependent variables are expanded as $n_\alpha = 1 + \sum_{j=1}^{\infty}\epsilon^j n_{\alpha j}$, $(v_x, v_y) = \sum_{j=1}^{\infty}\epsilon^{1+j/2}\,(v_{xj}, v_{yj})\,, (v_z, \phi) = \sum_{j=1}^{\infty}\epsilon^j\,(v_{zj}, \phi_j)$. Here the transverse velocity components (v_x, v_y) appear at higher order of ϵ than that of the parallel component v_z. This anisotropy is introduced by the influence of strong magnetic field. In this approximation the ion gyromotion is treated as higher order effect. By inserting these expressions into Eqs. (1)-(5) and collecting the terms in different powers of ϵ, we obtain in the lowest order, $n_{i1} = (l_z/M)^2\,\phi_1$, $n_{e1} = 3\phi_1$, $n_{p1} = -3\delta^{-2/3}\phi_1$,

$v_{z1} = (l_z/M)\,\phi_1$, $v_{x1} = v_{y1} = 0$ together with the dispersion law

$$M = \pm l_z \sqrt{\frac{(1-\delta)}{3(1+\alpha)}}, \tag{7}$$

where $\alpha \equiv \delta^{1/3}$. The flow may be outward or inward depending on the sign we consider in Eq. (5). Moreover, $M < 1$, i.e., the QIAWs propagate with the phase speed smaller than the ion-acoustic speed, and M increases as the positron to electron density ratio increases.

Next, the coefficient of ϵ^2 vanishes leading to $Q\phi_1^2 = 0$ where $Q = 1/\alpha - 9(1+\alpha)/(1-\delta)$. Without loss of generality we may assume that $\epsilon\phi_1 \to -\phi_0/2$ as $r \equiv (x, y, z) \to \infty$ so that $\phi_2 \to 0$ as $r \to \infty$. Thus, Q should be at least of the order of ϵ, i.e, $Q\phi_1^2 \sim \epsilon^3$ and is to be added to the third order contribution from Eq. (5). This gives a favorable condition for DL shocks instead of solitons. Physically, under this condition the free as well as the trapped particles are assumed to adjust themselves rapidly in order to maintain the quasineutrality at any time on each side of the propagating shocks. Now, the vanishing of the coefficients of ϵ^3 gives five equations. When $\partial\phi_3/\partial\xi$, $\partial n_{i3}/\partial\xi$, $\partial v_{z3}/\partial\xi$ etc. are eliminated and the first-order quantities are inserted into the resulting equation, terms containing ϕ_2 and ϕ_3 cancel, and the following MKdV equation is obtained ($\phi \cong \epsilon\phi_1$).

$$\frac{\partial^3\phi}{\partial\xi^3} = \lambda\frac{\partial\phi}{\partial\tau} + 2\mu\phi\frac{\partial\phi}{\partial\xi} + 3\gamma\phi^2\frac{\partial\phi}{\partial\xi}, \tag{8}$$

where $\lambda = P/S$, $\mu = Q/S$, $\gamma = R/S$ with $P = 6(1+\alpha)/l_z$, $R = 135(1+\alpha)^3/(1-\delta)^2 - 3(1-\delta)/2\delta$ and $S = 1 + (1-\delta)(1-l_z^2)/\omega_c^2 - 9H^2(1+\alpha)/4\alpha$. An asymptotic shock solution of Eq. (8) can be obtained as [33]

$$\phi(\xi,\tau) = -\frac{\phi_0}{2}\tanh\left(-Ns_1 + \sqrt{-\frac{\gamma}{8}}\phi_0\zeta\right) \tag{9}$$

where $\zeta = \xi - V\tau$, s_1 is a constant and $N-1$ represents the number of solitary waves in front of the shocks. Let us first investigate analytically the coefficients λ, μ and γ with the system parameters. We see that for $\delta < 1$, P is always negative and $Q(\sim\epsilon) \gtrless 0$ according as $0 < \delta < 0.001$ or $0.001 < \delta < 1$. Also, R is negative for $0 < \delta < 0.0065$ and positive otherwise. Moreover, $S \gtrless 0$ according as $H \gtrless H_c$, where H_c is the critical value of H given by

$$H_c = \frac{2}{3}\sqrt{\frac{\alpha}{1+\alpha}\left[1 + \frac{(1-\delta)(1-l_z^2)}{\omega_c^2}\right]}. \tag{10}$$

Note that this critical value, which depends parametrically on the density ratio δ, the obliqueness parameter l_z and the ion-cyclotron frequency ω_c gives a critical value of the electron density. The smaller the values of H_c the larger are the electron number densities. In order to consider the smaller values of H_c or higher densities, one might have to disregard the charge separation effect (the unity in the square brackets) in strongly magnetized plasmas ($\omega_c < 1$). In this case the critical value scales as $H_c \sim 2\sqrt{\alpha/(1+\alpha)}/3$. In a weakly magnetized case, $H_c \sim 2\sqrt{\alpha(1-\delta)(1-l_z^2)/(1+\alpha)}/3\omega_c$, which can relatively be larger than the strongly magnetized case. However, in both the cases one has to be careful about the particle density range in which the Fermi thermal speed is much smaller than the speed of light in vacuum ($V_{F\alpha} << c$) and the coupling parameter satisfies the relation: $g_Q \equiv 2m_e e^2/\varepsilon_0\hbar^2(3\pi^2\sqrt{n_0})^{2/3} \lesssim 1$ (this corresponds to the case where the quantum collective and mean field effects are important). Since $Q \sim \epsilon$ for $\delta \sim 0.001$, we consider the regime $0 < \delta < 0.0065$ in which $R < 0$. Thus, in order that the DL solutions exist we must have $H > H_c$ such that $S > 0$, since γ is to be negative.

Note that since S is always positive for δ, $l_z < 1$ and for $H = 0$, the DL solutions still exist in the absence of H. This implies that quantum effects will be relevant in dense plasma environments where electrons and positrons are degenerate (e.g., in magnetars as well in the next generation laser solid-density plasma interaction experiments) for which classical fluid model fails to describe the plasma dynamics. Basically, the quantum parameter H $(> H_c)$ restricts here the particle density to be of the order of $10^{34}\mathrm{m}^{-3}$ or lower (since higher values of H corresponds to lower density regimes) in order that the ion-acoustic DLs exist. Furthermore, the DLs are compressive or rarefactive according as $\delta \lesseqgtr 0.001$. Inspecting Eq. (9) one finds that while the width decreases, the amplitude of the stationary DLs increases with a slight increase of the density ratio δ. Also, the width increases, with the frequency ω_c and the obliqueness l_z.

Next, we numerically investigate Eq. (8) using Runge-Kutta scheme with an initial condition $\phi(\xi,0) = -(\phi_0/2)\tanh\left[\sqrt{-\gamma/8}\,(\phi_0/N)\,\xi\right]$ and with $\phi_0 \sim -0.05$, $N = 3$, $\delta \sim 0.001$, $H \sim 0.22$. The numerical values of the coefficients in Eq. (8) are $\lambda = -25.5$, $\mu = 0.17$ and $\gamma = -2.6 \times 10^3$. Note here that though the initial condition looks very similar to the asymptotic solution (9), but the constant N appears in different manner (For detailed discussion see, e.g., Ref. [33]). We use 1000 grid points with the system scale size $L_\xi = 100$, and choose the pulse size $L_p(\sim 1.2)$ to be less than L_ξ in order that the shock solutions exist. The shock profiles are shown in Fig. 1 giving a spatial scaling constant $(3.7\lambda_F)^{-1}$ for the initial profile and $(1.2\lambda_F)^{-1}$ for the asymptotic shock. The solitary waves in front of the shocks are ordered so that one with the maximum amplitude is nearest the shock. Initially the shock profile steepens, as time goes on it reaches a steady state with two solitary wave fronts (in the case of $N = 3$).

In conclusion, the formation of forward propagating DL shocks is possible in a strongly magnetized quantum e-p-i plasma. Such shocks composed of the compressive

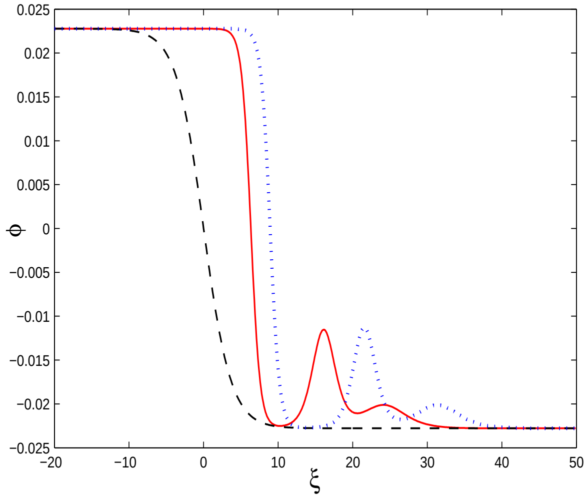

FIG. 1: (Color online) The profiles of the DL shocks for three different times. The initial profile ($\tau = 0$) steepens (dashed line), and simultaneously two solitary waves form in front of the shock (see the solid line for $\tau = 150$ and the dotted line for $\tau = 200$).

as well as rarefactive slow-wave fronts propagate with the speed less than that of the ion-acoustic speed. Furthermore, the DLs exist in dense plasma environments with particle density of the order of $10^{34}\mathrm{m}^{-3}$ or lower when the background electron population is much larger than that of positrons, and electron-positron annihilation is negligible [30]. Existence of such DL shocks may have significant role for the particle acceleration in the next generation laser solid-density plasma interaction experiments [5] as well as in compact astrophysical objects as evident from the recent observations in magnetized white dwarfs [14]. However, conclusive evidence needs further investigation in this area. In this way one may extend our investigation by considering the relativistic as well as the spin quantum effects [34] in a quantum MHD model, which, we hope, will give better understanding for the existence and properties of such novel DL structures. Future research is also expected to reveal other interesting applications of the shock solutions demonstrated here.

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